Book Reviews

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Discontinuous Finite Elements in Fluid Dynamics and Heat Transfer

B. Q. Li, edited by K.-J. Bathe, Springer, New York, 2006, 587 pp., $129.00

DOI: 10.2514/1.30646

THIS book is part of the Springer Series, *Computational Fluid and Solid Mechanics*. It has 12 chapters, each containing a section of exercises and chapter references, plus an index. The preface states it to be a monograph, written at an introductory level. The terminology "discontinuous finite element" denotes a specific implementation of what the literature terms the discontinuous Galerkin (DG) method. The DG methods developed in this book are computationally implemented using finite element (FE) spatial discretizations, hereafter referred to as DGFE.

The first seven chapters develop the formulation aspects of the DGFE algorithm, with the remainder presenting specific applications. After summarizing motivation and background, Chapter 2 details the base DGFE method for a linear two-point boundary value problem. The claimed advantage of coupling occurring only in the flux boundary integrals is substantially compromised by the clearly stated disadvantages of DGFE:

1) The theory requires resolving a second-order differential equation into two first-order equations, thereby doubling (quadrupling!) the degrees of freedom (DOF) in the resultant one-dimensional (three-dimensional) matrix expression.

2) Stabilization is a required input to prevent element matrices from becoming singular for pure diffusion problem statements. This rather ad hoc addendum is never invoked when using classical discrete methods.

Chapter 3 presents standard FE methodology for linear elliptic boundary value problems spanning up to three dimensions, including hierarchical elements and numerical quadrature, material that is standard fare in all FE text books. Section 3.4 summarizes the well-established theory on FE basis interpolation error, using language definitely not introductory in presenting aspects of Hilbert space and the usual array of norms. To its credit, Eqs. (3.123)–(3.127) present the standard FE basis asymptotic error estimates, in the limit mesh measure $h \Rightarrow 0$, for the class of piecewise continuous FE bases lying in H^p, which requires the corresponding p-differentiability be square integrable. The pertinence of this theory to the DGFE procedure is not detailed, nor is the degradation of this basic asymptotic error estimate due to solution or data nonsmoothness. These omissions render this exposure of questionable value in theoretical characterization of the *discontinuous* implementation process; in fact, attention to theoretical specifics is shallow throughout the book.

Chapter 4 details the DG linear basis FE implementation for steady and unsteady heat conduction in two dimensions, also potential flow. Clearly stated is the requirement to replace the Laplacian with two first-order partial differential equations, immediately tripling the DOF count in comparison to any classical discrete method. Computational stability accrues to appropriate evaluation of the DGFE theory-generated element boundary fluxes, with stable flux functions for both steady and unsteady heat conduction tabulated. Computational examples are given only for regular domains smoothly bounded by global coordinate surfaces, upon which homogeneous Dirichlet boundary conditions are specified, and appear generated using a single dense mesh of uniform triangles. These test case specifications are trivial, in my view, as the generated solutions are smoothly continuous. Why apply a discontinuous method to problem statements containing no nonsmooth content? A simple conductivity discontinuity would much better serve the requirement, and also would admit validation of an asymptotic error estimate. Recall that AIAA, the American Society of Mechanical Engineers, and international journal standards do not admit manuscript presentations based on single mesh solutions, which these appear to be.

The following chapter moves to scalar convection-diffusion, hence dispersion error. The development confirms that the DGFE method closely resembles classical finite volume procedures, the basis for which is readily interpreted as a non-Galerkin weak formulation requiring boundary flux approximations, generally finite difference generated. The chapter eventually introduces total variation diminishing (TVD) schemes, hence the slope and/or flux limiters which underlie the entire class of discontinuous, decades-old, flux-limited discrete methods, e.g., Godunov, Engquist-Osher, Lax-Friedrich, van Leer, and Roe with an "entropy fix." From this perspective, the DGFE algorithm clearly belongs to this class, but the author fails to prove, or even indicate, any benefit accruing to the DGFE formulation. The chapter ends with the essence (only) of a TVD, Runge–Kutta DG linear FE basis algorithm for (I assume) the Euler equations, with Fig. 5.18 presenting a sequence of refined mesh solutions for the classic double Mach reflection benchmark. This development really belongs, fully detailed, in Chapter 7, "Compressible Fluid Flows."

Chapter 6, "Incompressible Flows," is very short, leading off with repetition of the generic caution, "To ensure the numerical stability of the DGFE method, the numerical fluxes must be chosen carefully." After a multipage theory development for the primitive incompressible Navier–Stokes form, solutions are presented for the two-dimensional driven cavity at $Re = 50$, 100, and 1000, as generated on a uniform mesh of triangles. These appear as single mesh solutions devoid of solution adaptation, the graphics are exceptionally poor, and no comparison to well-known benchmark data is presented. The same criticism holds for the presented single uniform mesh solutions and graphics for the planar two-dimensional square thermal cavity benchmark for $10^4 \leq Ra \leq 5 \times 10^5$. Additionally, the $Ra = 5 \times 10^5$ solution does not exhibit the isotherm mirror symmetries well verified for the "correct" solution.

"Compressible Fluid Flows" is the title of Chapter 7, which leads off presenting the familiar hyperbolic conservation law forms for the Euler equations, the Rankine–Hugoniot relations, and the one-dimensional Riemann solver, both exact and approximate. TVD DG basis solutions are presented for pure unsteady convection of a square wave, for a mesh refinement study using constant (finite volume), linear, and quadratic FE basis implementations. Solution accuracy indeed improves with progression to more complete FE bases, but all solutions exhibit diffusion of the initial square wave. Results are also presented for the inviscid Burgers' equation and the classic shock tube. The chapter is completed with a discourse on the Euler hyperbolic conservation law form in multidimensions, an arbitrary Langrangian–Eulerian formulation, and then including viscous effects, all standard fare in computational fluid dynamics books. No DGFE computational results are presented for these specifications.

The remaining chapters cover special topics in radiative heat transfer, free and moving boundary problems, micro- and nanoscale thermal-fluid flow, and thermal flows in electromagnetic fields. From the modest example results detailed therein, it is obvious that the presented DGFE algorithm is capable of generating solutions for the associated conservation law statements. Of these presentations, only the DGFE application to the lattice-Boltzmann equation, and the last chapter presentation of an electroosmotic flow solution, represent an algorithmic challenge. And in all cases, the graphic presentations are substandard, apparently being grayscale renditions of what were originally color graphics. Finally, little data are presented to validate performance of the DGFE algorithm in comparison to either theory or experiment.

In summary, both the DG and classical Galerkin methods constitute a set of decisions on implementation of a weak formulation in seeking an approximate solution to a conservation principle set in the engineering sciences. The literature for classical Galerkin weak-form FE implementations in fluid-thermal applications is theoretically very rich, possessing numerous error estimates in H^p, both a priori and a posteriori, dependent on FE basis completeness and data nonsmoothness, and includes guidance on mesh adaptation for solution optimal accuracy. In total distinction, the DGFE theory base presented in this text appears at best quite heuristic, providing little guidance on performance quantification. Indeed, this "theory thinness," combined with omission of quality benchmark or validation data comparisons, I consider fundamental limitations on the true usefulness of the book.

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